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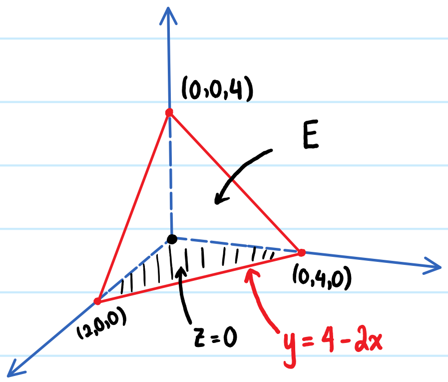
1) Find the volume of the tetrahedron enclosed by the planes $x=0, y=0, z=0$ and $2x+y+z=4$

Ans Recall that $V(E) = \iiint_E 1\, dV$

First draw the diagram E. First find the x, y and z-intercepts of the plane $2x+y+z=4$.

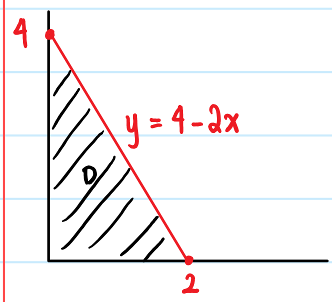


Since we are only interested in the plane in the first octant we have a sketch of E.

The lower boundary of the tetrahedron is $z=0$ and the upper boundary is the plane $2x+y+z=4$ i.e. $z = 4-2x-y$.

- Next, note that the planes $2x+y+z=4$ and $z=0$ intersect in the line $2x+y=4$ in the xy-plane. Therefore we can see that the projection of E onto the xy-plane, which we call D is given $\{(x,y) \mid 0 \le y \le 4-2x,\ 0 \le x \le 2\}$ (see Figure).

$$2x+y=4 \Downarrow y = 4-2x$$



Therefore,

$$E = \{(x,y,z) \mid 0 \le x \le 2,\ 0 \le y \le 4-2x,\ 0 \le z \le 4-2x-y\}$$

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Then,

$$V(E) = \int_0^2 \int_0^{4-2x} \int_0^{4-2x-y} 1 \, dz \, dy \, dx = \int_0^2 \int_0^{4-x} (4-2x-y) \, dy \, dx = \int_0^2 \left[4y - 2xy - \frac{y^2}{2} \right]_{y=0}^{4-2x} dx$$

$$= \int_0^2 4(4-2x) - 2x(4-2x) - \frac{1}{2}(4-2x)^2 \, dx = \int_0^2 (4-2x)(4-2x-2+x) \, dx$$

$$= \int_0^2 8 - 8x + 2x^2 \, dx = \left[8x - 4x^2 + \frac{2x^3}{3} \right]_0^2 = \frac{16}{3}$$